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Article

Analogy among Center of Gravity, Center of Mass, and Centroid of Rigid body: Analysis of Formula for Their Respective Calculation as per Configuration of Body and Utilization in Composite Application

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Abstract: The essential distinctions and similarities between the center of gravity, the center of mass, and the centroid of a rigid body in space are illustrated in this article. The use of an acceptable equation for a given configuration of the body situated at any location in a 3-D space is established and used in a composite application in the boiler lever safety valve.

Keywords: Centroid, the center of Gravity, Hand Lay-up Technique, Hair-Jute Hybrid Composite, Boiler Lever Safety Valve

1. Introduction

Gravity, or gravitation as it is frequently called, is the most common phenomenon that is extremely responsible for bringing physical or natural things towards one another. Gravitation acts as a force of attraction in all objects, including planets, stars, and galaxies. Although energy and mass are fundamental physical quantities, the gravitational force has an impact on them. The weight on Earth is also a result of gravity. The universe's basic gaseous ingredients condense to become stars, which then assemble into galaxies. As a result, huge objects in the universe are considered to bring the phenomena of gravity to light. Despite having an infinite range, the gravitational force's effect weakens as distance increases.

Albert Einstein introduced gravity in 1915, which is best exemplified by the general theory of relativity that describes gravity as the result of space-time curvature caused by uneven mass/energy distribution. A black hole is a well-known example of space-time curvature. Nothing, not even light, can escape a black hole after it has passed its event horizon [1]. At a lower gravitational potential, the larger gravitational force causes gravitational time dilation, in which time passes more slowly. Nonetheless, Newton's rule of universal gravitation, which says that an attractive force between two massive substances is exactly proportional to the product of their masses and inversely proportional to the square of the distance between them, is a good approximation of the idea of gravity.

The arithmetic mean ("average") position of all the points in a form is defined as the centroid or geometric center of a planar figure in mathematics and physics [2–6]. The centroid of any object in n-dimensional space is the mean position of all points in all coordinate directions, according to this definition. It is the point where an infinitesimally thin cut out of the shape may be properly balanced on the tip of a pin (assuming uniform density and a uniform gravitational field). While "barycenter" is a synonym for "centroid" in geometry, the barycenter is the center of mass of two or more bodies circling each other in astrophysics and astronomy. The arithmetic mean of all locations weighted by the local density or specific weight is the center of mass in physics. If a physical item has uniform density, its center of mass is the same as its shape's centroid.

The centroid of a radial projection of an area of the Earth's surface to sea level is referred to as the geographical center of that region in geography. The initial instance of gravity in the Universe, perhaps in the form of quantum gravity, supergravity, or a gravitational singularity, as well as conventional space and time, evolved during the Planck epoch (up to 1043 s after the creation of the Universe), presumably from a primordial condition, such as a false vacuum, quantum vacuum, or virtual particle, in an as yet unknown manner [2]. As a result, the quest for a theory of everything, the merger of the general theory of relativity and quantum mechanics (or quantum field theory) into quantum gravity, has become a study focus.

2. Results

2.1. Theory

The words 'the center of gravity,' 'the center of mass,' and 'Centroid' are similar in general, yet there are minor differences between these three concepts (Fig. 1). The center of Gravity is defined as the point through which the entire weight of the body acts or appears to be concentrated, whereas the center of Mass is defined as the point through which the entire mass of the body acts or appears to be concentrated. The Centroid is defined as the point at which the entire length, area, or volume of a body can be assumed to be concentrated.

A body's center of gravity is simply the consequence of gravitational forces acting on the particles that make up the body in any direction. It is denoted by the term center of gravity or simply gravity. It is the property of a body's gravitational influence.

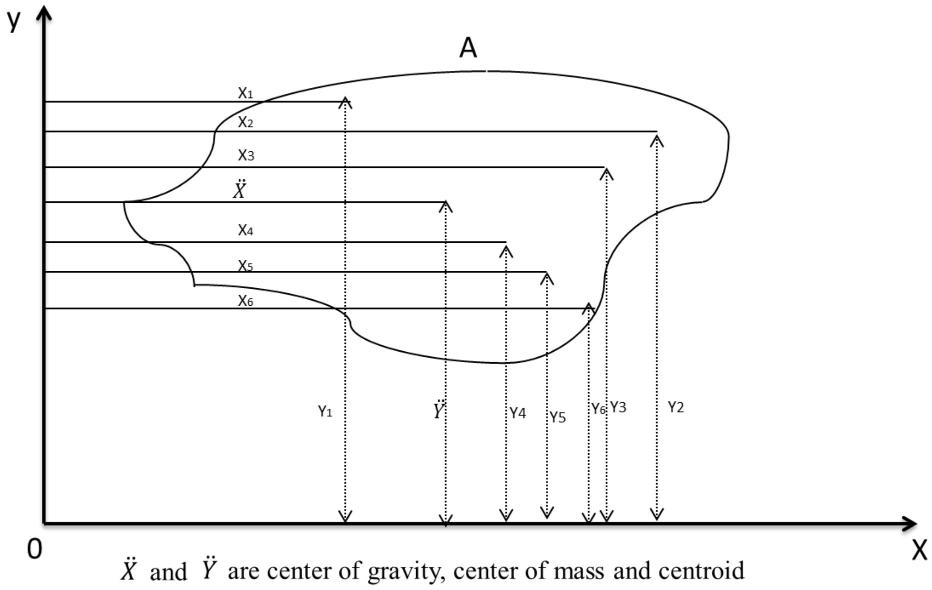

Fig. 1. Representation of the center of gravity, the center of mass, and centroid

Let us consider a rigid body 'A' placed in a space in the *x-y* plane. Body 'A' is composed of many particles weighing as W_i (where i=1,2,3,......) and let W_G be the weight of the entire body acting through its center of gravity.

Now using the Resultant of Force Law, we have

$$W_G = W_1 + W_2 + W_3 + W_4 + W_5 + ----------- + W_i \tag{1}$$

Now Varigon's Theorem of Moments is applied. Moment of resultant of any force system about a point is equal to the algebraic sum of the Moment of individual forces acting on a system about the same point.

Taking the moment about the *y*-axis,

$$(\ddot{X} \times W_G) = (X_1W_1 + X_2W_2 + X_3W_3 + X_4W_4 + X_5W_5 + --------- + X_iW_i) \tag{2}$$

$$\ddot{X} = \frac{(X_1W_1 + X_2W_2 + X_3W_3 + X_4W_4 + X_5W_5 + --------- + X_iW_i)}{(W_1 + W_2 + W_3 + W_4 + W_5 + --------- + W_i)} \tag{3}$$

$$\ddot{X} = \frac{\sum X_iW_i}{\sum W_i} \tag{4}$$

or,

$$\ddot{X} = \frac{\int X.dW}{\int dW} \text{(for continuous weight distribution)} \tag{5}$$

Similarly,

$$\ddot{Y} = \frac{\sum Y_i W_i}{\sum W_i} \tag{6}$$

or,

$$\ddot{Y} = \frac{\int Y.dW}{\int dW} \text{ (for continuous weight distribution)} \tag{7}$$

where $\ddot{X}$ and $\ddot{Y}$ are for the center of Gravity of body, and

$$W_G = W_1 + W_2 + W_3 + W_4 + W_5 + - - - - - - - - - - - + W_i \tag{8}$$

Now, the rigid body's center of mass is considered the sum of the individual masses of the particles that make up the rigid body. It is represented by COM and is the property of the mass of any rigid body in a constant gravitational field.

Thus, the center of gravity of a body is expressed as

$$\ddot{X} = \frac{\int X.dW}{\int dW} \text{ (for continuous weight distribution)} \tag{9}$$

As a well-known equation, $W = mg$, where, g is gravitational constant. Then, for continuous mass distribution,

$$\ddot{X} = \frac{\int x.dmg}{\int dmg} \tag{10}$$

$$\ddot{X} = \frac{g\int x.dm}{g\int dm} \tag{11}$$

$$\ddot{X} = \frac{\int x.dm}{\int dm} \tag{12}$$

$$\ddot{Y} = \frac{\int y.dm}{\int dm} \tag{13}$$

where $\ddot{X}$ and $\ddot{Y}$ are for the center of gravity of a body.

The centroid of a body refers to a point that overall represents the dimensions of the body. It is denoted by C and is the geometric property of the body. Considering the center of the mass of a body,

$$\ddot{X} = \frac{\int x.dm}{\int dm} \text{ (for continuous mass distribution)} \tag{14}$$

As a well-known equation,

$$m = p.V \text{ (p is the density, and V is the volume of a body).} \tag{15}$$

Then, for continuous mass distribution,

$$\ddot{X} = \frac{\int x.dpV}{\int dpV} \tag{16}$$

$$\ddot{X} = \frac{p\int x.dV}{p\int dV} \tag{17}$$

$$\ddot{X} = \frac{\int x.dV}{\int dV} \tag{18}$$

Similarly,

$$\ddot{Y} = \frac{\int y.dV}{\int dV} \tag{19}$$

where $\ddot{X}$ and $\ddot{Y}$ are for the center of gravity of a body, and

$$V = L.A \text{ (L is the length of the body).} \tag{20}$$

When A is the area of a body and for continuous mass distribution,

$$\ddot{X} = \frac{\int x.dV}{\int dV} \tag{21}$$

$$\ddot{X} = \frac{\int x.dLA}{\int dLA} \tag{22}$$

When L is constant and for continuous mass distribution,

$$\ddot{X} = \frac{L\int x.dA}{L\int dA} \tag{23}$$

$$\ddot{X} = \frac{\int x.dA}{\int dA} \tag{24}$$

Similarly, when A is constant, and $\ddot{X}$ and $\ddot{Y}$ are for the center of gravity of a body,

$$\ddot{X} = \frac{A\int x.dL}{A\int dL} \tag{25}$$

$$\ddot{X} = \frac{\int x.dL}{\int dL} \tag{26}$$

$$\ddot{Y} = \frac{\int y.dA}{\int dA} \tag{27}$$

$$\ddot{Y} = \frac{\int y.dL}{\int dL} \tag{28}$$

$$\ddot{Y} = \frac{\int y.dA}{\int dA} \tag{29}$$

for continuous mass distribution.

3. Discussion

Understanding this essential comparison between the center of Gravity, the center of Mass, and the Centroid assists in determining the stability of objects at rest and in motion. As a result, its use is found in a variety of areas, such as vehicles, structural projects, and industries. It may be used in boiler mountings in boiler plants. We used it in a boiler safety valve, specifically a lever safety valve made of a hair-jute hybrid composite [3]. In boilers, safety valves alleviate excess pressure and protect the boiler from exploding. A lever safety valve is a type of safety valve that consists of a valve body with a flange attached to the steam boiler. The valve is housed in the valve seat, which is made of bronze. The valve thrust is transmitted through the strut. The fulcrum of the lever is stationary, while the weight is linked to the opposite end. A guide is supplied to allow the lever to be moved vertically.

During normal pressure within the boiler, the lever safety valve stays in the seat. The strut's push caused the valve to remain in place until the steam pressure exceeded the safe operating pressure, as shown in Fig. 2(a). When the pressure within the boiler exceeds the safe operating pressure, the valve rises above the seat, allowing steam to escape outside the boiler (Fig. 2(b)).

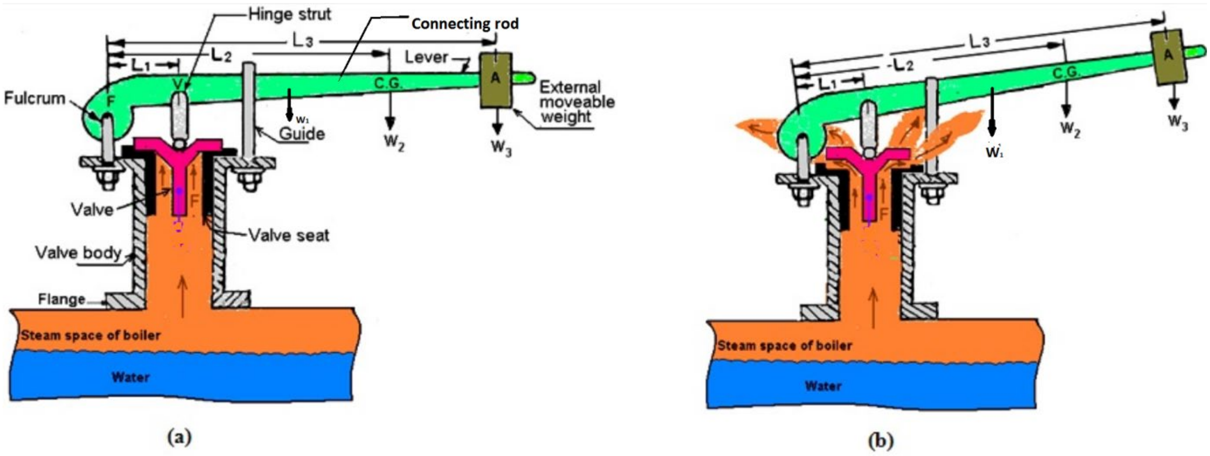

Fig. 2. (a) Normal steam pressure condition (b) Excess steam pressure condition

When the pressure within the boiler returns to normal, the valve returns to its regular position. When the thrust produced by the valve's strut exceeds the thrust applied by the steam pressure force, the valve rises. The thrust on the valve may be adjusted by adjusting the load at one end of the lever. The required weight W3 at the end of the lever for maintaining the safe working pressure in the boiler is obtained by taking moment about the hinged point F (fulcrum). Then,

$$F \times L_1 = W_3 \times L_3 \tag{30}$$

where, F = pressure force exerted by the pressurised steam

L_1 = distance of valve center from the hinged point

W_3 = weight attached at the other end

L_3 = distance of the weight from the hinged point

W_1 = weight of metal rod

However, the major problem that arises is that the metal rod that attaches the weights, strut. The hinged end possesses a certain weight which tends to bring the center of gravity of the combined metal rod and the attached weight (W_3) closer to the strut and reduce the effective emission of steam from the boiler. Therefore,

$$F \times L_1 = W_2 \times L_2 \tag{31}$$

where, F = pressure force exerted by the pressurised steam

L_1 = distance of valve center from the hinged point

W_2 = combined weight of the metal rod and attached weight (W3)

L_2 = distance of the C.G. of the combined rod-weight system from the F

Thus, the reduced moment ($W_2 \times L_2 - W_3 \times L_3$) leads to decreased steam emission. As a result, the excess pressurized steam cannot leave exerts stress on the bronze valve, which causes the valve to fail. To circumvent this difficulty, we used a rod made of lightweight material with excellent strength and durability. We used a rod formed of a hair-jute hybrid composite with 75 % hair and 25 % jute as reinforcing agents mixed with polyester as a matrix (Fig. 3). A composite is any product created by physically joining two or more existing materials, specified filler or reinforcing agent, and a suitable matrix binder.

The hand lay-up technology, which is the oldest, simplest, and largest fiber-reinforced plastic goods technique, is used to create the hair-jute hybrid composite (Fig. 3).

Fig. 3. Hair-jute hybrid composite with 75 percent hair and 25 percent jute

A mold with a flat surface, an activity, or an appositive form made of wood, metal, plastic, or a mixture of these materials. Fiber reinforcement and resin are manually applied to the mold surface. The perform is impregnated with a thermosetting liquid resin before being injected into the mold. The injection is performed using an applied pressure provided by an external source of high pressure or the use of a brush to spread the resin on the fiber before the mold is closed and the needed load is supplied to the mold.

Mechanical testing with the universal testing machine model 3039 following ASTM D3039-76 revealed that the 75 % human hair and 25% jute hybrid composite had a tensile strength of 23.5 MPa and impact strength of 4.25 MPa. This composite can be used to make the lever safety valve's connecting rod. This composite has a low specific weight, which results in increased specific strength and stiffness, as well as strong tensile strength, high impact strength, and superior thermal insulation qualities. It is also moisture resistant. Because this composite is much lighter in weight than the previously used metal rod, the center of gravity of the combined rod and external weight system is much closer to the attached weight end, i.e., close to L3, avoiding the reduction of the moment and facilitating the easy and fully relieving the escape of pressurized steam, which exerted relative less stress on the valve and thus increased its life. Furthermore, the composites may be employed in a variety of additional areas to bring the center of gravity of the system considerably closer to the ground level, increasing the system's stability. It may potentially be used to make frames for tiny load machines. As a consequence, the center of gravity is extremely significant in the creation and design of machine components.

4. Conclusions

In summary, the depiction of attributes of a rigid body's center of gravity, the center of Mass, and the centroid differs from one another. It is possible, however, that the center of gravity, the center of Mass, and the centroid of a symmetrical entity put in a uniform and equi-orientational gravitational field coincide. Furthermore, the center of gravity, the center of Mass, and the centroid with an axis of symmetry lie solely on that axis.

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